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On the Theory of Ion Currents to Probes at Low Pressures

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As is well known, Langmuir's theory of probe characteristics is based on the assumption that there is a space charge layer surrounding the probe. Outside this layer, the plasma is assumed to be quasi-neutral, the electric field zero, and the motion of particles random. The application of this theory to the ionic saturation current to a negatively charged probe resulted in considerable discrepancy with experiment. It has been shown recently[1,2] that the assumption of quasi-neutrality of the plasma outside the layer is possible only if outside the layer there exists a field which affects the motion of the ions. By taking this fact into account, Wenzl was able to formulate an improved theory of ionic saturation current to a negatively charged spherical probe and to compute for three special cases the distribution of the potential and the concentration of the charged particles in the vicinity of the probe. However, Wenzl's theory requires several graphical integrations of differential functions for each value of the probe potential and for each value of the discharge parameters; and it thus loses its practical value. As we shall show below, it is possible, by starting from premises similar to those in Wenzl's theory, to obtain a simple relation between the ionic current for large negative probe potentials and the electron current at the space potential.

When ions move in the field of attraction of a spherical probe, trapping occurs: i.e., for each given velocity, there is a sphere of radius r_0; once in this sphere, an ion then strikes the probe no matter what its original direction might have been. Therefore, inside the sphere r_0 there are no ions traveling away from the probe, and hence their motion may not be considered random. The potential of this sphere $\varphi(r_0)$ relative to the unperturbed plasma can generally be determined from the relation

$$-e\varphi(r_0) \sim kT_n, \tag{1}$$

where T_n is the ionic temperature. However, this condition also results[1] from the requirement that the plasma inside the region r_0 be quasi-neutral. We can assume that plasma remains quasi-neutral within the distance r_1 of the probe, and that thereafter the electron concentration begins to decrease sharply, i.e., up to the point where

$$-e\varphi(r_1) \approx kT_e, \tag{2}$$

where T_e is the electron temperature. The condition (2) coincides essentially with the criterion of the "sheath boundary" introduced by Boyd.[2] Theoretical

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considerations, as well as spectroscopic observations,[3] show that at low pressures

$$kT_a \ll kT_e \tag{3}$$

always; hence

$$r_0 > r_1. \tag{4}$$

The following basic conclusions are suggested by the above considerations:

1. Contrary to the Langmuir theory, in which it is assumed that the surface of the sheath is the ion collector, the surface of the radius r_0, whose area is larger, is the ion collector.

2. Whereas Langmuir assumed that ions reach the surface of the sheath with thermal velocities, the energies of the ions reaching that surface are of the order kT_e, as follows from the conditions (2) and (3).

3. Whereas Langmuir assumed the ion movement near the boundary of the sheath to be random, in view of conditions (2) and (3) we have much better grounds for assuming that the ions fall almost normally through the surface.

For the rest of the computation, we can keep the rough model of Langmuir's probe theory; that is, we can assume that for $r > r_1$ the concentrations of the charged particles equal their concentration in the unperturbed plasma, and that for $r < r_1$ the concentration of electrons equals zero. When allowance is made for the existence of a field outside the sheath, this model appears even justified, since the field compensates to a certain extent the effect of the probe, which attracts the ions. This is also confirmed by Wenzl's calculations.

Then, for the ionic current to the probe, we have

$$J = n_0 e u_1 S_1,$$

where $S_1 = 4\pi r_1^2$, and u_1 is the velocity of an ion at the distance r_1 from the probe.

Taking into consideration the fact that the electron current at the space potential I is expressed by the relation

$$I = \frac{n_0 e v_e S}{4},$$

where S is the area of the probe and v_e the average velocity of the electrons, we finally obtain

$$J = 4\sqrt{\frac{\pi m}{4M}}\frac{S_1}{S} I. \tag{5}$$

In order to eliminate the unknown quantity S_1, we must investigate the behavior of the potential for $r < r_1$. This amounts to solving Poisson's equation, taking as boundary conditions $\varphi = -\varphi_a$ on the surface of the probe, and $\varphi = 0$ and $(d\varphi)/(dr) = 0$ on the surface S_1.

The second condition is connected with the fact that for sufficiently large probe potentials $\varphi_a \gg kT_e$. The third condition follows from the fact that for sufficiently large negative probe potentials, the potential near the probe changes much more rapidly than it does near the surface S_1. This follows from Wenzl's computations. The calculation was thus reduced to Langmuir's problem for a spherical condenser, a problem to which Kan[4] found an exact solution. For simplicity, we shall limit ourselves to the case of practical interest $(r_1 - a) < a$, when the problem is approximately a plane-parallel one. In this case we can use the "three-halves law".

$$\frac{J}{S} = \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e}{M}}\,\frac{\varphi_a^{3/2}}{(r_1-a)^2}. \tag{6}$$

From (5) and (6) we obtain the following relation between the electron current at the space potential I and the ionic current for large negative potentials:

$$I = J\frac{\frac{1}{4}\left(\frac{4M}{\pi m}\right)^{1/2}}{\left[1+\frac{2}{3}\left(\frac{2e}{M}\right)^{1/4}\frac{\varphi_a^{3/4}}{\sqrt{J}}\right]^2}. \tag{7}$$

We find also the dependence of the thickness of the charged layer on the current and the potential of the probe:

$$\frac{r_1-a}{a} = \frac{2}{3}\left(\frac{2e}{M}\right)^{1/4}\frac{\varphi_a^{3/4}}{\sqrt{J}}. \tag{8}$$

For the case of a cylindrical probe, we again use (5) and (6), in which now $S = 2\pi al$ (where l is the length of the probe and a its radius), and $S_1 = 2\pi r_1 l$ (where r_1 is the radius of the cylindrical charged layer surrounding the probe). There results

$$I = J\frac{\frac{1}{4}\left(\frac{4M}{\pi m}\right)^{1/2}}{1+\frac{2}{3}\left(\frac{2e}{M}\right)^{1/4}\left(\frac{l}{2a}\right)^{1/2}\frac{\varphi_a^{3/4}}{\sqrt{J}}}; \tag{9}$$

$$\frac{r_1-a}{a} = \frac{2}{3}\left(\frac{2e}{M}\right)^{1/4}\left(\frac{l}{2a}\right)^{1/2}\frac{\varphi_a^{3/4}}{\sqrt{J}}. \tag{10}$$

The limits of applicability of (7) and (9) are subject, on the one hand, to the condition $e\varphi_a \gg kT_e$ and, on the other, to the requirement that the thickness of the charged layer should be smaller than the radius of the probe.

Table I.

I — $i = 100$ ma, $t = 11{,}2°$; II — $i = 1{,}0$ a, $t = 20{,}5°$; III — $i = 400$ ma, $t = 21{,}8°$

Spherical probe								Cylindrical probe			
I				II				III			
φ_a (volts)	J (μa)	$\frac{r_1-a}{c}$	I (ma)	φ_a (volts)	J (μa)	$\frac{r_1-a}{a}$	I (ma)	φ_a (volts)	J (μa)	$\frac{r_1-a}{a}$	I (ma)
103	60	0,91	2,8	106	155	0,59	10	103	330	0,78	32
69	50	0,75	2,8	79	130	0,51	10	83	310	0,69	31
40	40	0,56	2,8	55	110	0,41	9,5	63	280	0,59	30
24	30	0,44	2,5	36	90	0,35	8,5	43	250	0,48	29
19	20	0,45	1,6	17	60	0,25	6,5	23	192	0,33	23

These two conditions are usually met for spherical probes. For cylindrical probes of usual size, however, these requirements are not both met, the problem cannot be conceived as a plane one, and (6) must be replaced by the solution of Langmuir's problem for the cylindrical case.

Eqs. (7) and (9) were tested experimentally in a mercury discharge at low pressure in two discharge tubes of different types. In the first tube the middle part was formed into a globe of diameter 120 mm; in the center of the

globe there was a spherical platinum probe 2.3 mm in diameter. The second tube was a cylinder 32 mm in diameter along whose axis was fixed a cylindrical probe 2 mm in diameter and 8 mm long. A series of probe characteristics was taken for values of the discharge current i ranging from 50 ma to 1.3 amp and for vapor pressures corresponding to the temperatures t = 10 to 22 degrees. The ionic part of the characteristics was measured with negative potentials up to the order 115 v. Using (7) and (9), we determined the electron current to the probe at the space potential from the ionic part of the characteristic, and compared the result with the values for the current obtained from the electronic part of the characteristic. In every instance, the agreement was satisfactory; values of the electron current obtained from the ionic part of the probe characteristic were close to the values of the electron current obtained from the semilogarithmic curve at the departure from linearity. As an illustration, Table I gives the ionic part for the three characteristics and the values of the electron current (computed from the ion current by using Eqs. (7) to (10)) at the space potential and relative thickness of the sheath. The corresponding electronic parts of these characteristics are shown in Fig. 1, where the dashes indicate the values of I obtained from the ionic part of the characteristics, the calculated values being almost constant.

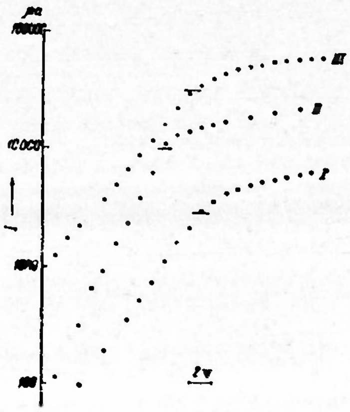

Fig. 1.

The sharp decrease of the values of I, computed at smaller negative potentials, is apparently associated with the impossibility of using the condition $e\varphi_a \gg kT_e$ in that part.

The method for obtaining the random electron current I from the ionic part of the characteristic is convenient because the lack of precision in determining the space potential hardly affects the result, whereas it does affect it in the usual method. All the above reasoning is valid for sufficiently low pressures, i.e., for $\lambda > r_0$. From the condition of quasi-neutrality of the plasma in the vicinity of r_0, it can be shown that

$$\frac{r_0}{r_1} \sim \left(\frac{T_e}{T_a}\right)^{1/4}.$$

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